# Striking the Right Balance: Why Standard Balance Tests Over-reject the Null, and How to Fix It

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#### **Abstract**

Economists often use balance tests to demonstrate that the treatment and control groups are comparable prior to an intervention. We show that typical implementations of balance tests have poor statistical properties. Pairwise t-tests leave it unclear how many rejections indicate overall imbalance. Omnibus tests of joint orthogonality, in which the treatment is regressed on all the baseline covariates, address this ambiguity but substantially over-reject the null hypothesis using the sampling-based p-values that are typical in the literature. This problem is exacerbated when the number of covariates is high compared to the number of observations. We examine the performance of alternative tests, and show that omnibus F-tests of joint orthogonality with randomization inference  $p$ -values have the correct size and reasonable power. We apply these tests to data from two prominent recent articles, where standard F-tests indicate imbalance, and show that the study arms are actually balanced when appropriate tests are used.

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## 1 Introduction

Balance tests are often used to demonstrate that treatment and control groups are comparable before treatment. Based on a review of all papers published in top five journals in economics between 2021 and 2023, we identified 69 randomized controlled trials (RCTs), out of which 62 (90%) discuss the results of balance tests (Table A.1 in Appendix).

Balance tests play a crucial role in establishing the comparability of the treatment and control groups in randomized controlled trials (RCTs) prior to any intervention. This is particularly important in RCTs where the randomization may not have actually been implemented correctly—for example, in situations where the research team lacks full control over the randomization process such as public lotteries (e.g. [Kerwin and Thornton](#page-36-0) [2021,](#page-36-0) [Gazeaud, Mvukiyehe, and Sterck](#page-35-0) [2023\)](#page-35-0). Balance tests are especially relevant when administrators may have incentives to manipulate the allocation process. Even in RCTs in which the research team is in charge of randomization, balance tests may be useful to demonstrate that randomization did not result in an unlucky draw, which can skew the results of the study [\(Leamer](#page-36-1) [1983\)](#page-36-1).<sup>[1](#page-1-0)</sup> Some scholars have argued in favor of re-randomizing if balance tests identify serious imbalances [\(Bruhn and McKenzie](#page-33-0) [2009\)](#page-33-0) although this can lead to complications for inference [\(Athey and Imbens](#page-32-0) [2017\)](#page-32-0). Balance tests also play an essential role in the analysis of natural experiments, to demonstrate that compared groups are similar before the quasi-random intervention or shock [\(Siu, Sterck, and Rodgers](#page-38-0) [2023\)](#page-38-0).

Economists tend to use two methods to assess balance, often together. First, 82% of the papers we reviewed use pairwise  $t$ -tests (or groupwise  $F$ -tests if there are more than two study arms) with a series of baseline variables and argue that treatment and control groups are balanced if few tests reject the null hypotheses at conventional thresholds. Normalized differences are sometimes reported alongside t-tests to show that any differences are small in size. Second, 32% of papers use omnibus tests of joint orthogonality, in which the treatment dummy is regressed on the full list of baseline covariates, and conclude that experimental groups are balanced if the test statistic is below a conventional significance threshold. Most papers reporting the  $p$ -value of an omnibus  $F$ -test of joint orthogonality

<span id="page-1-0"></span><sup>1</sup>See [Mutz, Pemantle, and Pham](#page-37-0) [\(2019\)](#page-37-0) for arguments against this practice.

use an OLS regression with robust standard errors to address heteroskedasticity in the linear probability model (LPM), or cluster-robust standard errors to account for clustered randomization.

In this paper, we use simulations to show that both of these approaches have poor statistical properties in term of size and power. Simple pairwise or groupwise t-tests and F-tests for individual variables pose different statistical challenges. With pairwise or groupwise t-tests, it is unclear how many rejections should lead to the conclusion that there is a balance problem or a randomization failure. Authors are left to subjectively assess whether an excessive number of tests have been rejected or whether one or more tstatistics are unreasonably large. One approach that is sometimes used is "vote counting", in which authors conclude that there is imbalance if e.g. more than 10 percent of tests reject the null at the 10% level. This approach is known to have low power in metaanalyses [\(Hedges and Olkin](#page-36-2) [1980\)](#page-36-2). We show that it also has incorrect size: it rejects the null at very high rates. This happens because the fraction of  $p$ -values below 0.10 is itself a random variable. Even for independent tests with the correct size it is centered at 10% under the null, and so is greater than  $10\%$  nearly half the time.<sup>[2](#page-2-0)</sup>

The omnibus tests of joint orthogonality typically deployed in the literature, which use sampling-based inference, have the incorrect size: they substantially over-reject the null hypothesis. This problem is worst when many baseline variables are included in the test and when heteroskedasticity-robust standard errors are used. This over-rejection of the null means that omnibus tests of joint orthogonality wrongly indicate imbalance issues where none are present. The over-rejection problem is very large under realistic conditions. For example, with 500 observations and 50 covariates that are independent and normally distributed, robust omnibus F-tests of joint orthogonality reject the null hypotheses at  $\alpha = 0.10$  approximately 50% of the time, instead of the expected 10%.

We propose and compare three alternative methods to assess balance: (1) an omnibus F-test of joint orthogonality with randomization inference  $p$ -values, (2) the minimum sharpened q-value to adjust  $p$ -values from pairwise t-tests and thereby control the

<span id="page-2-0"></span><sup>2</sup>Asymptotically it exceeds 10% exactly half of the time. For datasets with finite samples, some fraction have a rejection rate of exactly 10%.

false discovery rate [\(Benjamini, Krieger, and Yekutieli](#page-33-1) [2006;](#page-33-1) [Anderson](#page-31-0) [2008\)](#page-31-0), and (3) a Kolmogorov–Smirnov test to assess whether  $p$ -values from pairwise t-tests are uniformly distributed. We compare the performance of these methods in terms of statistical size and power using simulations. We conclude that omnibus F-tests of joint orthogonality with randomization inference p-values exhibit excellent performance in terms of both statistical power and size, for both individual and cluster RCTs. Randomization inference is also the conceptually correct method for calculating  $F$ -test  $p$ -values for balance tests in RCTs, as the uncertainty comes from the randomization process and not from sampling variation [\(Abadie et al.](#page-31-1) [2020\)](#page-31-1). In individually randomized designs, the minimum sharpened  $q$ -value from pairwise t-tests also performs well, providing the best statistical power to detect few large imbalances (even though it is conservative in terms of empirical size). The other approaches we test have problems with size or statistical power, especially for clustered RCTs.

We therefore recommend assessing balance using omnibus  $F$ -tests of joint orthogonality with randomization inference as this method is more reliable and flexible with different datasets, and it offers a more intuitive justification than alternative methods. This method can be complemented with the minimum sharpened  $q$ -value from pairwise t-tests if treatment is randomized at the individual level. We discuss how to implement these tests with multiple treatments as well.

To illustrate the value of our research, we re-assess the balance of two RCTs whose results were recently published in top-five journals [\(Garbiras-Díaz and Montenegro](#page-35-1) [2022,](#page-35-1) [Auriol et al.](#page-32-1) [2020\)](#page-32-1). With pairwise *t*-tests and vote counting,  $6\%$  and  $17\%$  of tests are rejected at the 10% level in [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1) and [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) respectively, suggesting possible balance issues in the latter RCT. If we use typical omnibus F-tests of joint orthogonality with sampling-based inference, we reject the null of overall balance for some treatments in both papers. However, we find no significant balance issues in either paper when using an appropriate omnibus  $F$ -test of joint orthogonality with randomization inference. These findings strengthen the internal validity of the two papers in question. They also have broader implications for the RCT literature:

while these two studies were not selected randomly, they are representative of the issues that affect balance tests in RCTs: 82% of recent randomized trials published in top five journals used pairwise t-tests or groupwise  $F$ -tests and vote counting and 32% used omnibus tests of joint orthogonality, and 100% of omnibus tests relied on sampling-based inference.

This paper contributes to three strands of the literature in empirical research in social science. First, it adds to existing work on the use of balance tests in randomized controlled trials. [Senn](#page-37-1) [\(1994\)](#page-37-1) prominently argued that balance tests should not be used at all; based on his work, using statistical tests to conclude that there are balance problems is commonly referred to as the "Table 1 fallacy", particularly in health research (e.g. [Sherry](#page-37-2) [et al.](#page-37-2) [2023\)](#page-37-2). In social science, however, balance tests are more widely supported. [Leamer](#page-36-1) [\(1983\)](#page-36-1) points out that unlucky draws in randomized trials can lead to exactly the same treatment assignments as would happen outside of an experiment, leading to the same concerns about a particular study's results being incorrect. Unbiasedness guarantees that those errors will cancel out on average, but offers no such promises about the results of any specific random assignment. In a similar vein, [Eckles](#page-35-2) [\(2021\)](#page-35-2) argues that balance tests are important for verifying that the randomization actually took place. Imperfect randomization and failure to comply with treatment assignment are common in social science RCTs.<sup>[3](#page-4-0)</sup> Our results help to clarify how to test for aggregate balance problems of the sort emphasized by [Eckles.](#page-35-2) An individual  $t$ -test or comparison of standardized differences is sufficient for seeing whether there is imbalance on a specific variable, as in [Leamer.](#page-36-1) To know whether the randomization protocol may have been violated, however, overall balance tests are needed.

We extend this literature by comparing the performance of various tests for both individually randomized and clustered designs, considering a wide range of sample sizes and number of covariates, and assess both the statistical size and power of the tests. We also build on work that uses randomization inference for testing overall balance. [Hansen and Bowers](#page-35-3) [\(2008\)](#page-35-3) develop an overall balance test and show that it performs well

<span id="page-4-0"></span><sup>&</sup>lt;sup>3</sup>For example, in the Perry Preschool program, some children were reassigned to different treatment statuses [\(Heckman, Pinto, and Shaikh](#page-36-3) [2024\)](#page-36-3).

when p-values are constructed using randomization inference. Their test is uncommon in economics. We show that the sampling inference-based  $F$ -tests that most economics papers actually use over-reject the null, but have the correct size and high power when randomization inference is used instead

Second, our paper contributes to the literature on methods for the design and analysis of randomized trials. [Bruhn and McKenzie](#page-33-0) [\(2009\)](#page-33-0) use simulations to show how to optimize balance in the design of RCTs and how to analyze the data conditional on specific designs, and [Athey and Imbens](#page-32-0) [\(2017\)](#page-32-0) provide an overall guide to the analysis of data from randomized experiments. [Abadie et al.](#page-31-1) [\(2020\)](#page-31-1) discuss how to correctly conduct inference for data-generating processes like RCTs where the uncertainty comes from the assignment process rather than random sampling. A related line of work discusses the value of pre-specifying one's plan for analyzing the data from RCTs ahead of time [\(Casey,](#page-34-0) [Glennerster, and Miguel](#page-34-0) [2012\)](#page-34-0). We build on this body of work by providing specific guidance on how to conduct tests for overall balance. Omnibus tests of joint orthogonality have been used in applied research for some time, and is suggested by [McKenzie](#page-36-4) [\(2015\)](#page-36-4). But there is no existing evidence on how to do inference on the F-statistics from this test. We show, in line with [Abadie et al.,](#page-31-1) that the  $p$ -values for these omnibus  $F$ -tests of joint orthogonality should properly be constructed using randomization inference.

Third, we contribute to an extensive body of research on the validity of empirical work in economics. [Brodeur et al.](#page-33-2)  $(2016)$  find that there is substantial "missing mass" in the distribution of test statistics, suggesting that researchers are engaging in  $p$ -hacking in the vein of [Simmons, Nelson, and Simonsohn](#page-37-3) [\(2011\)](#page-37-3). [Eble, Boone, and Elbourne](#page-35-4) [\(2017\)](#page-35-4) assess randomized trials in economics by the standards used in medical research, showing that there is substantial risk of bias in the reporting of economics RCTs. [Young](#page-38-1) [\(2019\)](#page-38-1) shows that statistical analyses of data from RCTs over-reject the null hypothesis due to the inappropriate use of sampling- (rather than design-) based inference. A related line of work shows that multiple testing problems mean that many RCT results are false positives [\(Anderson](#page-31-0) [2008\)](#page-31-0). In contrast with this previous work, we show that randomized experiments typically perform *better* than the literature might suggest: sampling-based

approaches to inference skew the results toward (incorrectly) rejecting the null. However, our findings also suggest the possibility of another sort of selective reporting. Since the standard omnibus test of joint orthogonality rejects the null so often, authors may be running it but not reporting it. Our suggested alternative approach, which uses randomization inference instead, can help head off this issue.

#### <span id="page-6-1"></span>2 Methods to assess test size and power

We use simulations with four different data generating processes (DGP) to assess the size and power of different omnibus tests of joint orthogonality. We conduct all our simulations using Stata. Stata code for our simulations is available in the replication package for our paper.

#### 2.1 Test Size

In statistics, the size of a test refers to the probability of erroneously rejecting the null hypothesis – in other words, the likelihood of committing a Type I error. Scholars distinguish the nominal size of a test, which is the threshold set by the researcher for the maximum allowable probability of a Type I error, from its empirical size, which is the actual observed rate of Type I errors when the test is applied to a large number of datasets. Statisticians typically try to construct tests for which the empirical size is equal to the nominal size. If the empirical size of a test is less than its nominal size, the test is said to be more conservative, meaning the actual rate of Type I errors is lower than the prespecified level. This is not problematic but may indicate that a more-powerful test is possible [\(Fisher and Robbins](#page-35-5) [2019\)](#page-35-5). However, a statistical test should be avoided if its empirical size of a test is greater than its nominal size, as this means there is an increased likelihood of Type I error, i.e. false positives.

In balance tests, the null hypothesis is typically that the study arms have equal means. If two groups differ only because of random chance,<sup>[4](#page-6-0)</sup> then a balance test will have correct

<span id="page-6-0"></span><sup>4</sup>For example, due to sampling variation or assignment variation, which implies that the means are equal in expectation but different for any particular realizations of the sampling or randomization process.

size if its p-values are uniformly distributed, rejecting the null hypothesis at the  $x$  percent level in x percent of realizations. The test is conservative if  $p$ -values are skewed to the left, leading to a reduced risk of Type I error. By contrast, the test size is problematic if p-values are skewed to the right, leading to an over-rejection of the null hypothesis and increased likelihood of Type I error.

We consider four DGPs to assess the size of balance tests. The first two DGPs use simulated data. We consider a simple DGP with  $N$  observations,  $n$  independent variables that are normally distributed  $\sim N(0, 1),$ <sup>[5](#page-7-0)</sup> and an independent treatment that is randomly assigned to half of the  $N$  observations (DGP 1). We also consider a more complex clustered design, in which the N observations are split into 100 clusters of equal size, the  $n$ variables are correlated within clusters (average intra-class correlation=0.2), and treatment is randomly assigned to half of the clusters (DGP 2). In our benchmark estimates, we assess test size by generating 500 simulated datasets with  $N = 500$  observations and  $n = 50$  covariates. To examine how test size varies with the number of regressors and sample size, we also vary the number of variables  $n$  from 10 to 100 and the number of observations N from 200 to 5000.

The two other DGPs use original datasets from existing randomized controlled trials but randomly (re-)assign the treatment variable using the same assignment rule as in the original paper. We selected two papers from our review of recent RCTs published in top five journals.

These papers were selected intentionally, rather than at random, because we wanted to explore key features of the DGP that were shown to be problematic in our simulations. First, we wanted to select one individually randomized trial and one cluster randomized trial, as the results of our simulations show that balance tests perform differently in these two types of RCTs. Then, we selected the individually randomized trial of [Garbiras-Díaz](#page-35-1) [and Montenegro](#page-35-1) [\(2022\)](#page-35-1), which has a relatively small sample size and large number of covariates; our simulations show that this combination exacerbates the spurious imbalance

<span id="page-7-0"></span> $5\text{As}$  a robustness check, we also considered a DGP with variables that are distributed following uniform, chi-squared, or binary distributions. Results suggest the distribution of variables has only minor impact on test size.

issues with traditional omnibus F-tests of joint orthogonality. We selected the cluster randomized trial of [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) because one of the omnibus F-test of joint orthogonality reported in the paper has a  $p$ -value below 0.1, and we suspected that this was not a genuine imbalance problem but rather an issue with the test.

In [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1), the authors study whether crowdsourcing the monitoring of elections is effective in combating electoral fraud. The intervention leverages a Facebook advertisement campaign aimed at encouraging citizens to report electoral irregularities during the 2019 mayoral elections in Colombia. A total of 698 municipalities (more than half the municipalities in the country) were randomized to one of three interventions or a placebo control group, resulting in four main study arms. Citizens in the placebo control group received a basic message reminding them about the date of the coming elections. For citizens in the "information" study arm, the message also advertised the website where irregularities can be reported, together with a web link. Citizens in the "call-to-action" study arm received a message with a call to action to report irregularities and act against them. Citizens in the "information  $+$  call-to-action" study arm received both the message containing the link to report irregularities as well as the call to action.<sup>[6](#page-8-0)</sup> In our simulations, we randomly re-assign the 698 municipalities into a "placebo" treatment arm or a control group and use balance tests with 33 baseline covariates to compare observations in both groups (DGP 3).

[Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) examine the hypothesis that insurance can be a motive for religious donations, and that believers give money to churches hoping to receive insurance against future economic shocks. The authors conduct a two-stage lab-in-the-field experiment where they randomly assign participants to a formal and commercially available funeral insurance policy. Then, using a dictator game, researchers measure participants' willingness to contribute money to the church and two other charitable recipients.

Participants were recruited from different branches of a well-established Pentecostal church and grouped into sessions. Randomization was done at the session level. At

<span id="page-8-0"></span><sup>&</sup>lt;sup>6</sup>In order to understand how candidates responded to the reporting campaign, the researchers additionally cross-randomized whether a candidate for mayor (or any of their staff) received a letter informing them about the reporting campaign. For simplicity and brevity, we do not consider this additional level of randomization here.

the start of each session, one participant per group was invited to pick one out of three unmarked envelopes to assign the group to the "Insurance", "Insurance Information" or "No Insurance" group. The group of participants who received the paper with the insurance label received a funeral insurance policy offered by a leading micro insurer active in the Ghanaian market. Groups assigned to the "Insurance Information" received only information about the insurance policy. All groups were then invited to play the dictator game. Participants who received the "No Insurance" envelopes were not assigned to the policy and did not have any sort of discussion about insurance.

In our simulations, we randomly re-assign the clusters into three groups and use balance tests with 10 covariates to compare observations in two of the groups (DGP 4).

We use the four DGPs to assess the size of balance tests. For each DGP and each balance test, we generate 500 datasets and run the test separately in each dataset. We then estimate the cumulative distribution of  $p$ -values, i.e. the proportion of  $p$ -values (out of 500) that are below a threshold t, for t ranging from 0 to 1 in steps of 0.01. By construction, the treatment indicators are independent from the baseline covariates. Therefore, the balance tests have the correct size if p-values are uniformly distributed on  $[0,1]$ , implying that the cumulative distribution of p-values is aligned with the 45 degree line. In contrast, balance tests over-reject the null hypothesis if the distribution of  $p$ values is skewed to the right and the cumulative distribution of  $p$ -values is above the 45 degree line for p-values below critical thresholds, and under-reject the null hypothesis if the opposite holds. When interpreting the results of simulations, we will often consider DGPs 1 and 3 together, as they are both individually randomized designs, and DGPs 3 and 4 together, as they are both clustered designs.

#### 2.2 Statistical Power

The power of a test is the probability that the test correctly rejects the null hypothesis when a specific alternative hypothesis is true. The power of a test is equal to  $1 - \beta$  where  $\beta$  is the probability of committing a type II error by wrongly failing to reject the null hypothesis.

To assess the power of balance tests, we add imbalances to some of the covariates of DGPs 1 and 2 and estimate how frequently the tests reject the null hypothesis that there is no imbalance between the treatment and control groups. We use the significance level of 10%, which is the higher threshold typically reported in economics.

We consider four approaches to generate imbalances of different magnitudes in different subsets of covariates. First, for one variable only, a very large imbalance is created by adding 0.25 to treated observations. Second, large imbalances in 10% of variables are created by adding 0.2 to treated observations. Third, for 20% of variables, medium imbalances are created by adding 0.15 to treated observations. Finally, small imbalances are created in 50% of variables by adding 0.1 to treated observations.

We consider simulated datasets with different numbers of observations, letting  $N$  ranging between 200 and 5,000. We estimate the proportion of  $p$ -values below 0.10. In our benchmark estimates, we consider simulated datasets with 50 covariates. As a robustness check, we also assess how statistical power changes when the number of covariates varies between 10 and 100.

When assessing statistical power, we focus on tests whose empirical sizes are equal or below their nominal sizes. Indeed, assessing the power of a test is misleading if its empirical size is above its nominal size, as a higher rate of rejecting the null hypothesis would likely be caused by more type I errors instead of fewer type II errors. If two balance tests have empirical size equal or below the nominal size, one should prefer the test with the highest statistical power to detect imbalance. On the contrary, if two tests have similar statistical power, one should prefer the test with the lowest empirical size, i.e. the lowest probability of type I error.

#### 3 Balance tests in economics

To gain insights into how researchers approach balance tests in the economics literature, we systematically reviewed all papers that appeared when searching for the words "experiment", "field experiment", "field-experiment" , "randomized controlled trial", "randomized controlled trials", "randomised controlled trial", and "random" in the search engine of each of the top five journals in economics: the American Economic Review, Econometrica, the Journal of Political Economy, the Review of Economic Studies, and the Quarterly Journal of Economics. We also reviewed papers in the American Economic Journal: Applied Economics given the journal's focus on applied research and particularly RCTs. We identified 69 papers that were published between 2021 and 2023 and report original results from a randomized controlled trial. We analyzed these papers to identify whether and how the authors conducted balance tests. Our findings are summarized in [Table A.1.](#page-53-0)

We find that 90% of the reviewed papers use balance tests, which we define as tests assessing the statistical significance or magnitude of the correlation between treatment status and a vector of baseline covariates. Our review of the literature shows that economists employ a variety of balance tests. Pairwise t-tests are reported in 65% of the papers, followed by omnibus  $F$ -tests or chi-squared tests of joint orthogonality  $(32\%)$ , groupwise  $F$ -tests (26%), and pairwise normalized differences (6.5%).

We investigate the statistical properties of these tests by categorizing them into two distinct groups based on whether they consider baseline covariates individually or jointly. Pairwise and groupwise tests examine the association between treatment status and the different baseline covariates, each considered independently. By contrast, omnibus tests of joint orthogonality consider the different baseline covariates jointly in a multivariate regression framework. These two categories of tests yield distinct statistical insights and challenges.

#### <span id="page-11-0"></span>3.1 Pairwise and groupwise tests

Pairwise t-tests are by far the most-frequently used method to test for balance. Pairwise t-tests are testing the null hypothesis of equality of means in the treatment and control groups for each baseline covariate considered separately. t-test difference can also be estimated by regressing covariates on the treatment status. Regressions give authors the advantage of controlling for fixed effects and clustering at the randomization level. When there are more than two treatment arms, pairwise t-tests are sometimes replaced by groupwise  $F$ -tests obtained by regressing each baseline covariate on the full vector of treatment indicators and then testing the joint equality of the coefficient estimates with zero. Groupwise F-tests should not be confused with omnibus tests of joint orthogonality, where instead the treatment indicator is regressed on the full vector of baseline covariates; we discuss these in [Section 3.2.](#page-13-0)

Pairwise t-tests and groupwise F-tests have the same underlying logic: for each variable, they test whether the differences across arms are consistent with what one would expect from random chance, under the null hypothesis. Among the 69 papers we reviewed,  $82\%$  report the results of pairwise t-tests or similar groupwise F-tests.

To complement significance tests, authors sometimes also report pairwise normalized differences [\(Imbens and Rubin](#page-36-5) [2015\)](#page-36-5). This happens in 6.5% of papers . The normalized difference between study arms is the difference in means divided by the pooled standard deviation  $(\sqrt{\sigma_C^2 + \sigma_T^2})$ . This is normally compared against some cutoff value such as 0.15 or 0.25. Large normalized differences suggest imbalances between study arms that are substantively large compared with the sample variance, which implies that there could be meaningful bias in estimated treatment effects as well. The value of this approach is that it avoids failing to reject the null simply because of a small sample.

Pairwise and groupwise balance tests are problematic for three related reasons. First, there is no clear rule for determining how many rejections of the null in a balance table constitute a balance problem. Second, this creates additional "researcher degrees of freedom" [\(Simmons, Nelson, and Simonsohn](#page-37-3) [2011\)](#page-37-3) to present balance tables as showing or not showing a problem, depending on the authors' preferences and audience pressures. Researchers may want to argue that there are no balance problems in order to make it easier to publish their papers. Conversely, stakeholders with a vested interest in a program continuing may wish to sweep inconvenient null results under the rug by claiming that the randomization had problems. Third, because of the lack of guidance, researchers sometimes use *ad hoc* rules of thumb like "vote counting", in which the balance table indicates an overall balance problem if more than a specific fraction of pairwise tests rejects the null [\(Hedges and Olkin](#page-36-2) [1980\)](#page-36-2).

We examine the performance of pairwise *t*-tests and vote counting in Appendix Figure [A.1.](#page-39-0) For 500 repetitions of DGPs 1 and 2 respectively, we calculate the proportion of ttests' p-values that are below versus above 0.1, as economists doing vote counting typically use this threshold to conclude that the study arms are balanced.[7](#page-13-1) We find that vote counting dramatically over-rejects the null hypothesis that treatment and control groups are balanced. In 37% of the datasets generated using DGP 1, strictly more than  $10\%$  of ttests are significant at the 10% level when considering a heteroskedasticity-robust variance estimator (the percentage is 34% with a variance estimator assuming homoskedasiticty). For DGP 2, 32% of datasets yield strictly more than 10% of *t*-tests that are significant at the 10% level when considering a cluster-robust variance estimator (the percentage is as high as  $64\%$  with a variance estimator assuming homoskedasiticty). These percentages are much higher than  $10\%$ , implying that vote counting – if applied strictly – would misleadingly lead researchers to over-estimate imbalance problems.

#### <span id="page-13-0"></span>3.2 Omnibus tests of joint orthogonality

Omnibus tests of joint orthogonality aim to address the limitations of pairwise and groupwise tests by considering baseline covariates jointly in a unique test yielding a unique statistic. Omnibus tests of joint orthogonality typically involve regressing the treatment dummy on the vector of baseline covariates and test the null hypothesis that all regression coefficients are jointly equal to zero. Among the papers we reviewed, 32% report the results of one or more omnibus tests of joint orthogonality.

Researchers face different options when implementing omnibus tests of joint orthogonality in practice, including the choice of methods for estimating regression coefficients and standard errors. Among papers employing an omnibus balance test, a large majority  $(86\%)$  report the F-statistic and associated p-value resulting from an Ordinary Least Squares (OLS) regression. A minority of studies (14%) opted for a chi-squared test resulting from a logit or probit regression, or their multinomial equivalent when there are more than two treatment arms.

<span id="page-13-1"></span> $7T$ o implement each  $t$ -test, we use the Stata command regress, with the options robust or cluster when relevant, and record the *p*-value of the regression coefficient.

Researchers also need to determine how to do inference on the test statistics to compute  $p$ -values. Among the papers we reviewed,  $100\%$  rely on sampling-based inference (we study randomization inference in [Section 4.1\)](#page-17-0). As the dependent variable in omnibus tests of joint orthogonality is either binary or categorical, heteroskedasticity is an issue when OLS is used to estimate a linear probability model (LPM). Statistical programs usually provide different heteroskedasticity-consistent (HC) variance estimators, including the HC1, HC2, and HC3 estimators [\(MacKinnon and White](#page-36-6) [1985\)](#page-36-6).<sup>[8](#page-14-0)</sup> Researchers also use cluster-robust variance estimators when treatment status is assigned at the cluster level. Among the reviewed papers that use omnibus tests of joint orthogonality, 4.5% used heteroskedasticity-robust estimators, 59% used cluster-robust estimators, and 27% used a variance estimator that assumes homoskedasticity.

In [Figure 1,](#page-15-0) we assess the size of omnibus F-tests and chi-squared tests of joint orthogonality using sampling-based inference.[9](#page-14-1) Our key finding is that all versions of the omnibus tests of joint orthogonality have empirical size above nominal size, rejecting the null hypothesis too frequently in some or all DGPs. When the DGP does not involve clustering (DGPs 1 and 3), correct test size is only obtained when the omnibus test is issued from an OLS regression with a variance estimator that (incorrectly) assumes homoskedasticity. All heteroskedasticity-robust variance estimators dramatically over-reject the null hypothesis. For instance, under DGP 1, an omnibus  $F$ -test of joint orthogonality resulting from an OLS regression with the robust HC1 variance estimator rejects the null hypothesis in 50% of samples, instead of the expected 10%. When a clustered design is considered (DGPs 2 and 4), all omnibus tests of joint orthogonality over-reject the null, even the one assuming homoskedasticity. The problem remains if cluster-robust standard errors are used.

<span id="page-14-0"></span><sup>8</sup>[Angrist and Pischke](#page-31-2) [\(2008\)](#page-31-2) point out that these estimators can have poor finite-sample performance, and also that they themselves have a sampling distribution and can be smaller than homoskedastic standard errors.

<span id="page-14-1"></span><sup>9</sup>The results using a probit model are very similar to those from the logit and hence are only reported in the Appendix (Figures [A.4](#page-42-0) and [A.5\)](#page-43-0).

#### <span id="page-15-0"></span>Figure 1 Size of Omnibus Tests of Joint Orthogonality Using Sampling-based Inference





(a) DGP 1: 50 independent covariates  $\sim N(0.1)$ 



(c) DGP 3: [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1)

*p*-value (d) DGP 4: [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1)

 $0.0 +$ 

0.0 0.2 0.4 0.6 0.8 1.0

0.2

*Notes:* The figures show the cumulative distribution of  $p$ -values from omnibus  $F$ -tests and chi-squared tests of joint orthogonality, using sampling-based inference. Each figure focuses on one data generating process (DGP) and is based on 500 simulated datasets. DGP 1 considers a data generating process with 500 observations, 50 independent variables that are normally distributed ∼N(0,1), and an independent treatment randomly assigned to half of observations. DGP 2 considers a data generating process with 500 observations split in 100 clusters of equal size, 50 variables that are normally distributed and correlated within clusters (average coefficient of intra-cluster correlation  $= 0.2$ ), and an independent treatment randomly assigned to half of the clusters. The data from [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1) includes 698 municipalities in Colombia, which are randomly re-assigned to a "placebo" treatment group or a control group; the tests use 33 baseline covariates and compare observations in the treatment and control groups. The data from [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) includes 1016 observations split in 148 clusters, with clusters randomly re-assigned to two "placebo" treatment arms and a control group; tests use 10 baseline covariates and compare observations in one treatment group and the control group. For each test, we estimate the proportion of  $p$ -values (out of 500) that are below a threshold  $t$ , for  $t$  ranging from 0.01 to 0.99 in steps of 0.01. We then use a fractional-polynomial prediction line to smooth the results. Figures [A.4](#page-42-0) and [A.5](#page-43-0) show how test size vary with the number of covariates and sample size for DGP 1 and DGP 2 respectively.

These problems are magnified when sample size is smaller and the number of covariates is larger. Figures [A.4](#page-42-0) and [A.5](#page-43-0) in the Appendix represent the proportion of tests that are rejected at the 10% level for different versions of omnibus tests of joint orthogonality, as a function of the number of observations and covariates. By construction, 10% of tests should reject the null if the test size is correct, as the treatment is randomly assigned and hence independent of the baseline covariates in all four DGPs. Under DGP 1, the size of omnibus F-tests of joint orthogonality assuming homoskedasticity is correct even when  $N$  is small and the number of covariates is large. The size of the other omnibus tests is correct only when N is very large ( $\approx$  5000) or when the number of covariates is low  $(n < 10)$ , but incorrect when the number of observations is small or moderate and the number of covariates is larger than 10. The HC2 and HC3 estimators perform no better, over-rejecting the null at comparable rates to HC1. Under DGP 2 (the clustered design), test size problems emerge even for large datasets and a relatively small number of covariates.

Overall, we conclude that the methods currently used by economists to assess covariate balance are generally inadequate. Pairwise and groupwise tests, while commonly used, rely on subjective assessments of multiple test results by researchers, introducing a degree of subjectivity and leaving room for interpretation. Although omnibus tests of joint orthogonality address these issues, they usually over-reject the null, both for simulated data and original data from existing RCTs.

### <span id="page-16-0"></span>4 Alternative methods

We examine three alternative approaches to assess balance: (1) omnibus tests of joint orthogonality with randomization inference,  $(2)$  sharpened q-values to adjust p-values from pairwise t-tests and thereby control the false discovery rate [\(Benjamini, Krieger, and](#page-33-1) [Yekutieli](#page-33-1) [2006;](#page-33-1) [Anderson](#page-31-0) [2008\)](#page-31-0), and (3) a Kolmogorov–Smirnov test to assess whether p-values from pairwise t-tests are uniformly distributed. We first describe the intuition for these three methods and then examine their properties in terms of test size and statistical

power.

## <span id="page-17-0"></span>4.1 Omnibus test of joint orthogonality with randomization inference

Omnibus tests of joint orthogonality can be used with randomization inference instead of sampling-based inference, as proposed by [Hansen and Bowers](#page-35-3) [\(2008\)](#page-35-3). Randomization inference involves comparing the observed test statistic with the theoretical distribution of the test, which is computed by re-estimating the test statistic for a random sample of all possible treatment assignment vectors.<sup>[10](#page-17-1)</sup> In this paper, we use 500 random reassignments for each test. The intuition for using this approach is that the uncertainty in randomized experiments comes not from *sampling* variation but from *assignment* variation differences across repetitions of the experiment in terms of which units are assigned to treatment versus control [\(Abadie et al.](#page-31-1) [2020\)](#page-31-1). Since the randomized "treatments" do nothing, the sharp null hypothesis of a zero treatment effect for all observations is true by construction. We reject this null hypothesis if the observed test statistic is at the extreme of the estimated theoretical distribution—for example, beyond the 90<sup>th</sup>, 95<sup>th</sup>, or  $99<sup>th</sup>$  percentile.<sup>[11](#page-17-2)</sup>

Randomization inference aligns well with the intuition of balance tests, which examine the uncertainty or variation resulting from the randomization process and not from sampling. Another positive aspect of randomization inference is that it does not require specifying a model of the error term, which typically depends on a set of unknown parameters. This makes randomization inference more robust to certain non-normality or heteroskedasticity violations [\(Young](#page-38-1) [2019\)](#page-38-1).

<span id="page-17-1"></span><sup>&</sup>lt;sup>10</sup>The exact theoretical distribution of the test can in theory be obtained by estimating the test statistic for all possible treatment assignment vectors. However, this is computationally infeasible for all but the smallest samples.

<span id="page-17-2"></span><sup>&</sup>lt;sup>11</sup>We need not consider the lower tail of the probability distribution because  $F$ -statistics are weakly positive by construction.

#### 4.2 Adjustments for multiple hypothesis testing

Pairwise t-tests and groupwise F-tests are problematic because multiple tests are used to test one hypothesis, which is that the treatment arms are balanced. This issue could in principle be addressed using methods that adjust  $p$ -values to account for multiple hypothesis testing.

Two main approaches to address multiple hypothesis testing have been proposed in the literature [\(Anderson](#page-31-0) [2008\)](#page-31-0). A first group of corrections aim to control for the Familywise Error Rate (FWER), which is the probability of making at least one type I error among all the hypotheses being tested. The goal is to control this probability at a desired significance level. The second group of corrections aim to control for the False Discovery Rate (FDR), which the expected proportion of false discoveries (type I errors) among the rejected hypotheses. If all null hypotheses are true, then FWER and FDR are equivalent [\(Anderson](#page-31-0) [2008,](#page-31-0) p. 1487). This equivalence is important as, with pairwise t-tests and groupwise F-tests and balance tests more generally, null hypotheses are expected to be true.

Both FWER and FDR corrections can therefore be considered in the context of balance tests. In Appendix Figures [A.2](#page-40-0) and [A.3,](#page-41-0) we compare Romano-Wolf stepdown  $p$ -values [\(Romano and Wolf](#page-37-4) [2005;](#page-37-4) [Clarke, Romano, and Wolf](#page-34-1) [2020\)](#page-34-1), which control the FWER, and sharpened  $q$ -values, which control the FDR [\(Anderson](#page-31-0) [2008\)](#page-31-0). The figures show that both categories of corrections have similar statistical size and power for individually randomized designs (DGPs 1 and 3) but only the FDR correction has an empirical test size below its nominal size for clustered designs (DGPs 2 and 4), while the FWER correction substantially over-rejects the null.

In what follows, we therefore focus on sharpened q-values, which control the FDR. Controlling the FDR at level q implies imposing that the proportion of type I errors is below q. The basic method for this approach, from [Benjamini and Hochberg](#page-33-3) [\(1995\)](#page-33-3), is the following. Select a critical value for the test  $p_{crit}$ . Sort the p-values in increasing order and count them; call the total number  $M$ . Each has a rank,  $r$ , from 1 (the smallest) to  $M$  (the largest). Then, starting from the largest p-value, we test each one against  $p_{crit} \times (r/M)$ . So if there are 10 p-values then the largest is tested against 0.10, the second-largest against 0.09, and so forth. We stop when we get to the first rejection and reject all tests with smaller p-values.

The approach we use augments this method in three ways. First, we "sharpen"  $q_{crit}$ to improve statistical power while still controlling the FDR at the same rate, following [Benjamini, Krieger, and Yekutieli](#page-33-1) [\(2006\)](#page-33-1). Second, we use [Anderson](#page-31-0) [\(2008\)](#page-31-0)'s approach to compute not just whether a test was rejected at the  $q_{crit}$  level but the smallest  $q_{crit}$  for which the test would be rejected, which can be interpreted in the same way as a standard  $p$ -value.<sup>[12](#page-19-0)</sup> Third, we use the minimum of all the sharpened  $q$ -values as a test for overall balance. When we apply the q-value procedure to the  $p$ -values of M pairwise t-tests, we obtain  $M$  test statistics. While this helps determine which variables are imbalanced, it does not provide a unique determination of whether there is overall imbalance. We do this by rejecting the null hypothesis that treatment arms are balanced if the minimum sharpened q-value is below a conventional significance threshold (usually 0.1 in economics). This is equivalent to rejecting the null if any q-value is less than the threshold.

#### 4.3 Kolmogorov–Smirnov test

If a treatment is randomly assigned, then the  $p$ -values of pairwise  $t$ -tests should be uniformly distributed. This can be tested using a Kolmogorov–Smirnov (K–S) test, which is nonparametric statistical test that can be used to compare a sample distribution with a known reference distribution.

In the context of balance tests, the null hypothesis of the K-S test is that the sample of p-values from pairwise t-tests comes from a uniform distribution. The K–S statistic quantifies the maximum vertical distance between the empirical distribution of  $p$ -values and a uniform distribution. A key limitation of the K–S test is that it has low statistical power, especially for small sample sizes [\(Razali and Wah](#page-37-5) [2011\)](#page-37-5).

<span id="page-19-0"></span><sup>&</sup>lt;sup>12</sup>One potential limitation of this approach is that it technically only works for independent tests (in Anderson's simulations it also works for positively dependent tests). Thus we may expect it to work better in DGP 1, where the covariates are independent, as compared with the other DGPs that do have a non-zero correlation structure.

#### 4.4 Test Size

We assess the size of these alternative balance tests in [Figure 2,](#page-21-0) considering the four DGPs described in [Section 2.](#page-6-1) For each method, we consider variance estimators assuming homoskedasticity and either heteroskedasticity-robust or cluster-robust variance estimators, depending on whether the treatment is allocated to individuals or clusters. We consider individually randomized designs (DGPs 1 and 3) and clustered designs (DGPs 2 and 4) separately.

For DGP 1, all of the tests we consider have an empirical size equal to or below the nominal size. For DGP 3 however, only the omnibus  $F$ -tests with randomization inference and the minimum q-values from pairwise t-tests with a variance estimator assuming homoskedasticity have an empirical test size at or below the nominal size. We compare their statistical power in the next section. By contrast, minimum  $q$ -values with a HC1 variance estimator and both versions of the Kolmogorov–Smirnov test have incorrect sizes, over-rejecting the null hypothesis of balance.

In DGPs that mimic a clustered RCT (DGPs 2 and 4), empirical size is equal to or below the nominal size only for the omnibus F-tests with randomization-based inference and minimum  $q$ -values from pairwise t-tests with cluster-robust variance estimators. We compare the statistical power of these tests in the next section. By contrast, minimum q-values with a variance estimator assuming homoskedasticity and both versions of the Kolmogorov–Smirnov tests tend to over-reject the null hypothesis.

These results are confirmed when varying both sample size and the number of co-variates (Appendix Tables [A.6](#page-44-0) and [A.7\)](#page-45-0). In all DGPs, minimum  $q$ -values appear to be conservative for high significance thresholds, indicating the tests tend to generate fewer type I errors than expected. This is not an issue *per se*, but it may indicate that a more powerful test could be designed [\(Fisher and Robbins](#page-35-5) [2019\)](#page-35-5).

#### <span id="page-21-1"></span>Figure 2

<span id="page-21-0"></span>Size of Omnibus Tests of Joint Orthogonality Using Randomization Inference, Kolmogorov–Smirnov Tests, and Minimum  $q$ -values



(a) DGP 1: 50 independent covariates  $\sim N(0,1)$ 



(c) DGP 3: [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1)





*Notes:* Each plot shows the cumulative distribution of p-values/minimum q-values for one data generating process (DGP), based on 500 simulated datasets. DGP 1 has 500 observations, 50 independent variables that are normally distributed  $\sim N(0,1)$ , and an independent treatment randomly assigned to half of observations. DGP 2 has 500 observations split in 100 clusters of equal size, with 50 variables that are normally distributed and correlated within clusters (average  $ICC = 0.2$ ), and an independent treatment randomly assigned to half of the clusters. [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1) includes 698 municipalities in Colombia, which we randomly re-assign to a "placebo" treatment group or a control group; the tests use 33 baseline covariates. [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) includes 1,016 observations split into 148 clusters, which we randomly re-assign to two "placebo" treatment arms and a control group; the tests use 10 baseline covariates and compare observations in one treatment group and the control group. Figures [A.6](#page-44-0) and [A.7](#page-45-0) show how test size varies with the number of covariates and sample size for DGP 1 and DGP 2 respectively.

#### 4.5 Statistical Power

We assess the power of the balance tests in [Figure 3](#page-23-0) for DGP 1 and [Figure 4](#page-24-0) for DGP 2. We focus on balance tests that have correct sizes to avoid cluttering figures with misleading information.

For DGP 1, which assumes normally distributed variables and no clustering, we find that minimum q-values offer the best statistical power when only one variable is imbalanced, while the power of omnibus F-tests of joint orthogonality with randomization inference is intermediate.<sup>[13](#page-22-0)</sup> Omnibus  $F$ -tests of joint orthogonality with randomization inference have higher power than minimum  $q$ -values when a larger number of imbalances that each have a smaller magnitude are considered. This suggests that the two approaches might be complementary in individually randomized RCTs.

With DGP 2, we find that omnibus F-tests of joint orthogonality using randomization inference and cluster-robust standard errors have the highest power, while minimum qvalues from pairwise t-tests with cluster-robust variance estimators largely fail to detect imbalance. The results are similar when we vary sample size and the number of covariates (Figures [A.8](#page-46-0) and [A.9](#page-47-0) in the Appendix). For clustered RCTs, we conclude that omnibus Ftests of joint orthogonality with randomization inference are a valid tool to assess balance, achieving the correct test size and higher statistical power than other approaches.

<span id="page-22-0"></span><sup>&</sup>lt;sup>13</sup>We do not show the results of Kolmogorov-Smirnov tests and minimum  $q$ -values with a HC1 variance estimator as these approaches have incorrect size in DGP 3. [Figure A.8](#page-46-0) shows that the statistical power of minimum q-values with a HC1 variance estimator is similar to that of the minimum  $q$ -values with a variance estimator assuming homoskedasticity. The statistical power of Kolmogorov-Smirnov tests is low.

<span id="page-23-0"></span>Figure 3 Power of Omnibus Tests of Joint Orthogonality for DGP 1 UsingRandomization Inference, Kolmogorov–Smirnov Tests, and Minimum <sup>q</sup>-values



 variance estimator and Kolmogorov-Smirnov tests are hidden as these approaches have incorrect test size (see [Figure](#page-21-1) 2). For each method, the light dots show the N-specific share of p-values or minimum q-values below 0.10, and the lines show fractional-polynomial predictions for  $N = 5000$   $E_{\text{O}}$  imbalanced by adding 0.25 to treated observations. In Panel (b), 10% of variables are made imbalanced by adding 0.2 to treated observations. In Panel (c), 20%。<br>lt:a<br>l l D<br>sl ws<br>rc <sup>1,000</sup><sup>2,000</sup><sup>3,000</sup><sup>3,000</sup><sup>4,000</sup><sup>5,0</sup><br>
(c) 20% imbalanced variables, 0.15 SD<br>
ts of simulations with a data generating process<br>
assigned to half of observations. Imbalances are<br>
atted observations. In Panel (b), 10% of  $\frac{1}{2}$ a hered $\frac{1}{2}$ d $\frac{q}{2}$ be <sup>1,000</sup><sup>2,000</sup><sup>3,000</sup><sup>3,000</sup><sup>4,000</sup><sup>5,0</sup><br>
(d) 50% imbalanced variables, 0.10 SD<br>
ating 50 independent variables that are normally<br>
en created for a subset of the *n* variables. In Pa<br>
e made imbalanced by adding 0.2 to tr Notes: The figures show the results of simulations with a data generating process creating 50 independent variables that are normally distributed ∼N(0,1) and an independent treatment randomly assigned to half of observations. Imbalances are then created for a subset of the *n* variables. In Panel (a), one variable is made of variables are made imbalanced by adding 0.15 to treated observations. In Panel (d) 50% of variables are imbalanced, by adding 0.1 to treated observations. Toproduce each figure, a total of 100 simulated datasets of N observations are generated, for each  $N \in \{200, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000\}$ .  $N \in [200, 5000]$ . [Figure](#page-46-1) A.8 shows how the power of the tests varies with the number of covariates and sample size. Results from minimum q-values with HC1

<span id="page-24-0"></span>Figure 4Power of Omnibus Tests of Joint Orthogonality for DGP 2. Using Randomization Inference, Kolmogorov–Smirnov Tests, and Minimum <sup>q</sup>-values



 these approaches have incorrect test size (see [Figure](#page-21-1) 2). below 0.10, and the lines show fractional-polynomial predictions for  $N \in [200, 5000]$ . [Figure](#page-47-1) A.9 shows how the power of the tests varies with the number of 100 clusters of equal size and <sup>50</sup> variables that are normally distributed and correlated within clusters (with an average ICC of 0.2). An independent treatment es 5 c<br>es 5 c<br>ad ut weser <sup>1,000</sup><sup>2,000</sup><sup>3,000</sup><sup>3,000</sup><sup>4,000</sup><sup>5,000</sup><sup>5,000</sup><br>
(c) 20% imbalanced variables, 0.15 SD<br>
sults of simulations with a data generating proce<br>
60 variables that are normally distributed and coclusters and zero otherwise. Im N, a total of 100 simulated datasets of N observations are generated. For each method, the light dots show the N-specific share of p-values or minimum q-values  $\frac{1}{2}$ Notes: The figures show the results of simulations with a data generating process with N observations (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ) that are split into o<br>it it ea<br>a<br>c<br> $\alpha$ <br> $0$ <br>it it it it is <sup>1,000</sup><sup>2,000</sup><sup>3,000</sup><sup>3,000</sup><sup>4,000</sup><sup>5,000</sup><sup>5,000</sup><br>
(d) 50% imbalanced variables, 0.10 SD<br>
th *N* observations (with  $N \in \{200, 500, 1000, 200$ <br>
ed within clusters (with an average ICC of 0.2).<br>
ated for a random subset of is equal to one in half of the clusters and zero otherwise. Imbalances are then created for <sup>a</sup> random subset of the <sup>n</sup> variables. In Panel (a), one variable is imbalanced, by adding 0.25 to treated observations. In Panel (b), 10% of variables are imbalanced, by adding 0.2 to treated observations. In Panel (c), 20% ofvariables are imbalanced, by adding 0.15 to treated observations. In Panel (d), 50% of variables are imbalanced, by adding 0.1 to treated observations. For eachcovariates and sample size. Results from minimum q-values with a variance estimator assuming homoskedasticity and Kolmogorov-Smirnov tests are hidden as

### 5 Multiple treatments and cross-randomization

Many randomized experiments involve more than one treatment, and thus more than two study arms. Out of the 69 RCTs that we identified in our literature review, 27 (39%) have more than one treatment. For these studies, we can examine balance between each treatment and the control group. That is the best course of action if we are concerned about the [Leamer](#page-36-1) [\(1983\)](#page-36-1) problem of unlucky random assignments. In that case, omnibus F-tests of joint orthogonality using randomization inference are the optimal approach.

For assessing the [Heckman, Pinto, and Shaikh](#page-36-3) [\(2024\)](#page-36-3) problem of violations of the randomization protocol, however, it is necessary to look for problems with overall balance. This means that we need to run combined omnibus tests of joint orthogonality across all study arms. Linear regression cannot handle this, but it is possible to implement a comparable test using multivariate analysis of variance and covariance (MANOVA) or multinomial logit with sampling-based or randomization inference. The minimum  $q$ value approach and other multiple-hypotheses adjustments can also be easily adapted by considering the p-values from pairwise t-tests for each baseline covariate and each possible comparison between treatment arms. We consider individually randomized designs (DGPs 1 and 3) and clustered designs (DGPs 2 and 4) separately.

For the individually randomized designs in DGPs 1 and 3, we show in [Figure A.10](#page-48-0) that all approaches have their empirical size equal or below the nominal size, except the multinomial logit with sampling-based inference, which over-rejects the null hypothesis.<sup>[14](#page-25-0)</sup> We therefore omit this approach in power tests. Of the four remaining tests, the minimum q-value has the best statistical power to detect one large imbalance, while MANOVA and the multinomial logit with randomization inference have higher statistical power to detect multiple small imbalances.

For DGPs 2 and 4, which are clustered designs, only MANOVA and multinomial logit with randomization inference have the correct test size, while minimum  $q$ -values are again conservative, under-rejecting the null [\(Figure A.10\)](#page-48-0). We focus on these approaches

<span id="page-25-0"></span><sup>&</sup>lt;sup>14</sup>The multinomial logit can fail to converge when the number of observations is low compared to the number of covariates [\(Figure A.13\)](#page-51-0).

in power tests and find that multinomial logit with randomization inference is, by far, the approach that has the best statistical power to detect single large or multiple small imbalances. We therefore recommend using this approach for clustered designs with multiple treatments. It is important to bear in mind, however, that this test is not high-powered: large sample sizes are needed to detect imbalances.

## 6 Revisiting existing papers

In this section, we reassess the balance of two RCTs whose results were recently published in top five journals.

#### 6.1 Garbiras et al. (2022)

To assess balance, [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1) report the results of pairwise t-tests, considering 33 baseline covariates (e.g. statistics on past reporting of irregularities, socioeconomic covariates, political covariates, and region dummies). They use "vote counting" to conclude that municipalities are well balanced across treatment arms, reporting that "Only 16 differences in means out of 264 comparisons in Table A2 are statistically significant at a 10 percent level or less." The authors do not report the results of omnibus tests of joint orthogonality.

In [Table 1,](#page-28-0) we report the results of the different tests of joint orthogonality discussed in our paper. With sampling based inference and a heteroskedasticity-robust variance estimator, we find that two out of five tests are statistically significant at the 10% level. Two other *p*-values are just above 0.1, which could raise concerns. However, these results may be misleading: Section [3.2](#page-13-0) concluded that such omnibus  $F$ -tests tend to over-reject the null hypothesis that groups have equal means. Indeed, if we use the randomization inference procedure that performs best in our simulations, we find that the  $p$ -values of omnibus F-tests of joint orthogonality are all well above 0.1. These results are consistent with our conclusion that these tests have correct test size when treatment is assigned at

the individual level. We reach the same conclusion using minimum  $q$ -values.<sup>[15](#page-27-0)</sup>

We also find that the control and treatment arms are well balanced when considering the multiple treatment arms together using MANOVA with sampling-based inference (pvalue  $= 0.41$ ), MANOVA with randomization inference (*p*-value  $= 0.41$ ), multinomial logit with randomization inference ( $p$ -value = 0.48), and the minimum sharpened  $q$ -value from pairwise t-tests (minimum  $q$ -value = 1).

<span id="page-27-0"></span><sup>&</sup>lt;sup>15</sup>All p-values of Kolmogorov-Smirnov tests are also above 0.1. However, we refrain from interpreting these results given the poor size and statistical power of this test.

		$F$ -test p-value	Min.	$K-S$		
Inference $=$	<b>SI</b>	<b>SI</b>	$\rm RI$	$q$ -value	$p$ -value	
	HC1/Cluster	H <sub>0</sub>	HC1/Cluster	HC1/Cluster	HC1/Cluster	
	$\left(1\right)$	$\left( 2\right)$	$\left(3\right)$	(4)	(5)	
Panel A: Replication of Garbiras-Díaz and Montenegro			(2022)			
Any treatment vs. Control	0.088	0.287	0.278	1.000	0.737	
Information vs. Control	0.155	0.366	0.562	0.413	0.595	
Call to actions vs. Control	0.065	0.417	0.446	1.000	0.643	
Info $+$ call to action vs. Control	0.126	0.404	0.528	1.000	0.105	
Any letter vs. No letter	0.428	0.631 0.664		1.000	0.349	
Panel B: Replication of Auriol et al. (2020)						
Insurance vs. all other arms	0.988	0.967	0.996	1.000	0.005	
Insurance info vs. all other arms	0.079	0.070	0.202	0.868	0.891	
No insurance vs. all other arms	0.068	0.055	0.186	1.000	0.811	

<span id="page-28-0"></span>Table 1Replication of Existing Papers

 Notes: This table presents the results of different balance tests using the datasets from Garbiras-Díaz and [Montenegro](#page-35-6) [\(2022\)](#page-35-6) and [Auriol](#page-32-2) et al. [\(2020\)](#page-32-2). We rely on the same set of baseline covariates and present the same sets of comparisons as used by the authors in their original papers. For [Auriol](#page-32-2) et al. ([2020\)](#page-32-2), wepresent the additional comparison "No insurance vs. all other arms".

#### 6.2 Auriol et al. (2020)

In Table II of the main paper, [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) present a table of balance tests with twelve preregistered covariates.<sup>[16](#page-29-0)</sup> They consider both pairwise  $t$ -tests and omnibus  $F$ -tests of joint orthogonality.<sup>[17](#page-29-1)</sup> Out of the 24 p-values of pairwise t-tests, 4 are statistically significant at the 10% threshold (17%). A researcher using simple "vote counting" would conclude that the study arms are imbalanced. However, Section [3.1](#page-11-0) showed that "vote counting" should be avoided as it leads to over-rejections of the null hypothesis.

In [Table 1,](#page-28-0) we present the results of different joint tests of balance. With samplingbased inference, two out of three omnibus  $F$ -tests of joint orthogonality are significant at the 10% level, both with a cluster-robust variance estimator and with a variance estimator assuming homoskedasiticty. Researchers using these tests may wrongly conclude that the RCT has a problem of imbalance. However, when we instead use randomization inference, all  $p$ -values are above 0.1. We obtain a similar conclusion with minimum  $q$ -values. Overall, these additional tests suggest there is no balance issue in the [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) RCT, contrary to the conclusions of conventional sampling-based inference.<sup>[18](#page-29-2)</sup>

We also conclude that the control and treatment arms are well balanced when considering the multiple treatment arms together using multinomial logit with randomization inference (*p*-value  $= 0.28$ ).

<span id="page-29-0"></span><sup>&</sup>lt;sup>16</sup>The covariates are age, gender, total monthly income, a dummy of the employment status, three indicator variables reflecting ethnic group membership (Akan, Ewe, or Ga), and indicators for daily church attendance, praying multiple times per day, attending the revival week, and being recruited in the second wave.

<span id="page-29-1"></span> $17$ Pairwise t-tests are estimated for the twelve baseline covariates and two types of comparison: the "Insurance" treatment arm versus the "Insurance Information" treatment arm, and the "Information insurance" treatment arm versus the "No Insurance" treatment arm. For the two omnibus F-tests of joint orthogonality, the authors consider slightly different comparison groups, as the "Insurance" treatment arm is compared to the two other groups together in the first  $F$ -test, and the "Insurance Information" treatment arm is compared to the two other groups together in the second test. The F-tests are also estimated using a reduced list of baseline covariates, dropping the dummy variables identifying revival weeks and the second wave. We are able to exactly replicate the balance test results presented by the authors for both the pairwise  $t$ -tests and omnibus  $F$ -tests of joint orthogonality. In our analysis, we consider the same comparisons and variables as the authors used in their F-tests of joint orthogonality.

<span id="page-29-2"></span><sup>&</sup>lt;sup>18</sup>We refrain from interpreting the results of Kolmogorov–Smirnov tests, which were shown to have poor test size and statistical power in Section [4.](#page-16-0)

## 7 Conclusion

The use of balance tests in randomized experiments remains controversial. They can be powerful tools to demonstrate that randomized experiments were actually conducted as designed and did not lead to study arm allocations that may yield misleading estimates. However, some statisticians argue that they are unhelpful, falsely implying problems where there are none.

Our paper shows that this problem is even worse than previously recognized. The usual method of testing for overall balance yields far too many rejections of the null hypothesis, so that even experiments with no balance problems whatsoever appear to have serious issues. We show that randomization inference-based omnibus balance tests perform much better than the conventional approach using sampling-based inference. The latter has a high probability of Type I error with a large number of covariates, while the former has no such issue with test size, and it has high statistical power.

Future work on balance tests should explore how these tests should best be used in practice. For example, what would happen if researchers abandoned all RCTs in which (correctly implemented) omnibus balance tests show an overall balance problem? And in particular, how does that vary with the true rate of randomization failures, and the bias caused by imperfect compliance with randomization protocols? If the true null hypothesis holds, and all RCTs were in fact correctly run, then throwing out studies imbalanced treatment allocations could cause treatment effect estimates to be biased on average. But if some experiments really are run incorrectly then throwing them out would reduce bias. Which pattern dominates is an empirical question, and one that should be informed by both careful simulations and engagement with the practitioners who actually implement these experiments in the field.

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## Online Appendix

<span id="page-39-0"></span>

mator (DGP 1)

(e) Pairwise t-tests, OLS with HC1 variance esti-(f) Pairwise t-tests, OLS with HO variance estimator (DGP 1)



(g) Pairwise t-tests, OLS with cluster-robust vari-(h) Pairwise t-tests, OLS with HO variance estiance estimator (DGP 2) mator (DGP 2)

#### Figure A.1

Pairwise *t*-tests for DGP 1 and DGP 2: distribution of share of p-values below 0.10.

Notes: Each figure is based on 500 simulated datasets and shows the cumulative distribution of the share of p-values from pairwise t-test that are statistically significant at the  $10\%$  threshold. DGP 1 considers a data generating process with 500 observations, 50 independent variables that are normally distributed  $~\sim N(0,1)$ , and an independent treatment randomly assigned to half of observations. DGP 2 considers a data generating process with 500 observations split in 100 clusters of equal size, 50 variables that are normally distributed and correlated within clusters (average coefficient of intra-cluster correlation  $= 0.2$ ), and an independent treatment randomly assigned to half of the clusters. For each DGP, 500 datasets are generated, and for each dataset, we estimate the p-values of the 50 pairwise t-tests and calculate the share of p-values that are below 0.1. The four figures shows the distribution of these shares for DGPs 1 and 2 and for heteroskedasticity-robust and cluster-robust variance estimators as well as for a variance estimator assuming homoskedasticity.

<span id="page-40-0"></span>Figure A.2 Size of Balance Tests Using FWER and FDR Multiple-Hypothesis Testing Adjustments



(a) DGP 1: 50 independent covariates  $\sim N(0,1)$ 





(c) DGP 3: data from [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1)

(d) DGP 4: data from [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1)

 $0.0 \neq$ 

0.0 0.2 0.4 0.6 0.8 1.0 *p*-value

*Notes:* Each figure shows the cumulative distribution of minimum p-values or minimum  $q$ -values from the tests in question. Each figure focuses on one data generating process (DGP) and is based on 500 simulated datasets. DGP 1 considers a data generating process with 500 observations, 50 independent variables that are normally distributed  $\sim N(0,1)$ , and an independent treatment randomly assigned to half of observations. DGP 2 considers a data generating process with 500 observations split in 100 clusters of equal size, 50 variables that are normally distributed and correlated within clusters (average coefficient of intra-cluster correlation  $= 0.2$ ), and an independent treatment randomly assigned to half of the clusters. The data from [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1) includes 698 municipalities in Colombia, which are randomly re-assigned to a "placebo" treatment group or a control group; the tests use 33 baseline covariates and compare observations in the treatment and control groups. The data from [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) includes 1,016 observations split into 148 clusters, with clusters randomly re-assigned to two "placebo" treatment arms and a control group; the tests use 10 baseline covariates and compare observations in one treatment group and the control group. Separately for each of the 500 simulated datasets and each method: (1) we use the Stata command rwolf2 to estimate n FWER-adjusted p-values and record the minimum of the p-values; and (2) we apply the method of [Anderson](#page-31-0) [\(2008\)](#page-31-0) to the n p-values from the pairwise t-tests to obtain n sharpened q-values and record the minimum of these sharpened q-values. For each method, we estimate the share of test statistics (out of 500) that are below a threshold  $t$ , for  $t$  ranging from 0.01 to 0.99 in steps of 0.01. We then use a fractional-polynomial prediction line to smooth the results.

<span id="page-41-0"></span>Figure A.3Power of Balance Tests Using FWER and FDR Multiple-Hypothesis Testing Adjustments for DGP 1



(c) 20% imbalanced variables, 0.15 SD

(d) 50% imbalanced variables, 0.10 SD

 the number of covariates and sample size.minimum of the p-values; and (2) we apply the method of [Anderson](#page-31-3) ([2008\)](#page-31-3) to the n p-values from the pairwise t-tests to obtain n sharpened q-values and record produce each figure, a total of 100 simulated datasets of N observations are generated, for each  $N \in \{200, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000\}$ method, fractional-polynomial prediction lines are used to predict the power of tests for  $N \in [200, 5000]$ . [Figure](#page-46-1) A.8 shows how the power of the tests varies with 0<sup>1,000</sup><sup>2,000</sup><sup>3,000</sup><sup>3,000</sup><sup>4,000</sup><sup>5,000</sup><sup>5,000</sup><br>
(c) 20% imbalanced variables, 0.15 SD<br>
sults of simulations with a data generating proces<br>
lly assigned to half of observations. Imbalances a<br>
reated observations. In Pan the minimum of these sharpened q-values. For each  $N$  and each method, the light dots on each figure show the share of test statistics below 0.10. For each  $N = 5000$   $5000$ ,  $\Gamma$ ; and  $\Gamma$  and  $\Gamma$  and  $\Gamma$  and  $\Gamma$  and Notes: The figures show the results of simulations with a data generating process creating 50 independent variables that are normally distributed  $\sim N(0,1)$  and an eadered)<br>contredd)<br>tapit<br>sf <sup>1,000</sup><sup>2,000</sup><sup>3,000</sup><sup>3,000</sup><sup>4,000</sup><sup>5,000</sup><br>
(d) 50% imbalanced variables, 0.10 SD<br>
ating 50 independent variables that are normally en<br>
en created for a subset of the *n* variables. In Pan<br>
e made imbalanced by adding 0.2 independent treatment randomly assigned to half of observations. Imbalances are then created for a subset of the *n* variables. In Panel (a), one variable is made imbalanced by adding 0.25 to treated observations. In Panel (b), 10% of variables are made imbalanced by adding 0.2 to treated observations. In Panel (c), 20% of variables are made imbalanced by adding 0.15 to treated observations. In Panel (d) 50% of variables are imbalanced, by adding 0.1 to treated observations. ToFor each method, each N, and each of the 100 simulated datasets, (1) we use the Stata command rwolf2 to estimate n FWER-adjusted p-values and record the



Figure A.4 Size of Omnibus Tests of Joint Orthogonality for DGP 1 Using Sampling-based Inference

<span id="page-42-0"></span>(e) Chi<sup>2</sup> -test, Logit with HO variance estimator (f) Chi<sup>2</sup> -test, Probit with HO variance estimator

*Notes:* The figures show the results of simulations with a data generating process creating  $n$  independent variables that are normally distributed  $\sim N(0,1)$  (with  $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ ) and an independent treatment randomly assigned to half of observations. Simulated datasets of N observations are generated (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ). Each figure focuses on one type of balance test and is constructed as follows. For each combination of  $n$  and  $N$ , 500 simulated datasets are created and the light dots show the share of tests'  $p$ -values below 0.10. For each  $N$ , fractional-polynomial prediction lines are then used to predict the size of tests for  $n \in [10, 100]$ .



Figure A.5 Size of Omnibus Tests of Joint Orthogonality for DGP 2 Using Sampling-based Inference

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0.0 0.2 0.4  $\rightarrow$  0.4 0.6  $0.0$  20.0  $40.0$  60.0 80.0 0.0 20.0 40.0 80.0 80.0 80.0 *N* (# of observations) *N* (# of observations) (c) F-test, Logit 0.2 0.4 0.6 0.0 20.0 40.0 60.0 80.0 (d) F-test, Probit

<span id="page-43-0"></span>Notes: The figures show the results of simulations with a data generating process with N observations (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ) that are split into 100 clusters of equal size, and n variables that are normally distributed and correlated within clusters (with  $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ , and an average intra-cluster correlation coefficient of 0.2). An independent treatment is equal to one in half of the clusters and zero otherwise. Each figure focuses on one type of balance test and is constructed as follows. For each combination of  $n$  and  $N$ , 500 simulated datasets are created and the light dots show the share of tests'  $p$ -values below 0.10. For each  $N$ , fractional-polynomial prediction lines are used to predict the size of tests for  $n \in [10, 100]$ .

Figure A.6 Size of Omnibus Tests of Joint Orthogonality for DGP 1 Using Randomization Inference, Kolmogorov–Smirnov Tests, and Minimum q-values



(a) F-test randomization inference, HO variance (b) F-test randomization inference, HC1 variance estimator estimator



<span id="page-44-0"></span>(e) Kolmogorov–Smirnov test, HO variance esti-(f) Kolmogorov–Smirnov test, HC1 variance estimator mator

Notes: The figures show the results of simulations with a data-generating process that creates n independent variables that are normally distributed  $\sim N(0,1)$  (with  $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ ) and an independent treatment randomly assigned to half of observations. Simulated datasets of N observations are generated (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ). For each combination of n and N, we create 500 simulated datasets and run pairwise t-tests for all  $n$  variables for each dataset. The Kolmogorov– Smirnov tests are run on the p-values of the pairwise t-tests. The minimum  $q$ -value is the minimum sharpened q-value obtained after applying the method of [Anderson](#page-31-0)  $(2008)$  to the p-values from the pairwise t-tests. For each  $N$ , the light dots show the *n*-specific share of test statistics below 0.10 and the lines show fractional-polynomial predictions for  $n \in [10, 100]$ .

Figure A.7 Size of Omnibus Tests of Joint Orthogonality for DGP 2 Using Randomization Inference, Kolmogorov–Smirnov Tests, and Minimum q-values



<span id="page-45-0"></span>Notes: The figures show the results of simulations with a data-generating process with N observations (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ) that are split into 100 clusters of equal size, and n variables that are normally distributed and correlated within clusters (with  $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ , and an average intra-cluster correlation coefficient of 0.2). An independent treatment is equal to one in half of the clusters and zero otherwise. For each combination of  $n$  and  $N$ , we create 500 simulated datasets and run pairwise t-tests for all  $n$  variables. The Kolmogorov–Smirnov tests are run on the  $p$ -values of the pairwise t-tests. The minimum  $q$ -value is the minimum sharpened  $q$ -value obtained after applying the method of [Anderson](#page-31-0) [\(2008\)](#page-31-0) to the p-values from the pairwise t-tests. The light dots show the share of p-values or minimum q-values below 0.10. For each  $N$ , fractional-polynomial prediction lines are used to predict the size of tests for  $n \in [10, 100]$ .

#### <span id="page-46-1"></span>Figure A.8

<span id="page-46-0"></span>Power of Omnibus Tests of Joint Orthogonality for DGP 1 Using Randomization inference, Kolmogorov–Smirnov Tests, and Minimum q-values



*Notes:* The figures show the results of simulations with  $n \in \{10, 25, 50, 100\}$  independent ∼N(0,1) variables, and an independent treatment randomly assigned to half of observations. Imbalances are then created for a random subset of the *n* variables. In Panels (a-c), one variable is made imbalanced by adding 0.25 to treated observations. In Panels (d-f), 10% of variables are made imbalanced by adding 0.2 to treated observations. In Panels (g-i), 20% of variables are made imbalanced by adding 0.15 to treated observations. In Panels (j-l), 50% of variables are made imbalanced by adding 0.1 to treated observations. Each figure focuses on one method and one type of imbalance and is constructed as follows. For each  $N \in \{200, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000\}$ , a total of 100 simulated datasets with n baseline covariates and N observations are generated. We run pairwise t-tests for all n variables in each dataset. The Kolmogorov–Smirnov tests are run on the  $p$ -values of pairwise t-tests. The minimum  $q$ -value is the minimum sharpened  $q$ -value obtained from applying the method of [Anderson](#page-31-0) [\(2008\)](#page-31-0) to the p-values from the pairwise t-tests. For each method and each  $n$ , the light dots show the N-specific share of p-values or minimum  $q$ -values below 0.10, and the lines show fractional-polynomial predictions for  $N \in [200, 5000]$ .

<span id="page-47-1"></span><span id="page-47-0"></span>Figure A.9 Power of Omnibus Tests of Joint Orthogonality for DGP 2 Using Randomization Inference, Kolmogorov–Smirnov Tests, and Minimum q-values



*Notes:* The figures show the results of simulations with  $N \in \{200, 500, 1000, 2000, 5000\}$  observations split into 100 clusters of equal size, and  $n \in \{10, 25, 50, 100 \text{ normally distributed variables that are correlated}$ within clusters (with an average intra-class correlation of 0.2). An independent treatment is equal to one in half of the clusters and zero otherwise. Imbalances are then created for a subset of the n variables. In Panels (a-c), one variable is made imbalanced, by adding 0.25 to treated observations. In Panels (d-f), 10% of variables are made imbalanced, by adding 0.2 to treated observations. In Panels (g-i), 20% of variables are made imbalanced, by adding 0.15 to treated observations. In Panels (j-l), 50% of variables are made imbalanced, by adding  $0.1$  to treated observations. For each  $N$ , we generate a total of 100 simulated datasets of N observations and run pairwise t-tests for all  $n$  variables in each dataset. The Kolmogorov–Smirnov tests are run on the p-values of the pairwise t-tests. The minimum  $q$ -value is the minimum sharpened q-value obtained after applying the method of [Anderson](#page-31-0) [\(2008\)](#page-31-0) to the  $p$ -values from the pairwise t-tests. For each method and each n, the light dots show the N-specific share of p-values or minimum q-values below 0.10, and the lines show fractional-polynomial predictions for  $N \in [200, 5000]$ .

<span id="page-48-0"></span>Figure A.10 Size of Omnibus Tests of Joint Orthogonality with Multiple Treatment Arms



(c) DGP 3: data from [Garbiras-Díaz and Mon](#page-35-1)[tenegro](#page-35-1) [\(2022\)](#page-35-1) (d) DGP 4: data from [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1)

*Notes:* The figures show the cumulative distribution of  $p$ -values and minimum  $q$ -values from omnibus tests of joint orthogonality, using sampling-based inference. Each figure focuses on one data generating process (DGP) and is based on 500 simulated datasets. DGP 1 considers a data generating process with 500 observations, 50 independent variables that are normally distributed  $\sim N(0,1)$ , and a categorical variable identifying four randomly-assigned treatment arms with one quarter of observations in each. DGP 2 considers a data generating process with 500 observations split in 100 clusters of equal size, 20 variables that are normally distributed and correlated within clusters (average coefficient of intra-cluster correlation  $= 0.2$ ), and a categorical variable identifying four randomly-assigned treatment arms with one quarter of observations in each. The data from [Garbiras-Díaz and Montenegro](#page-35-1) [\(2022\)](#page-35-1) includes 698 municipalities in Colombia, which are randomly re-assigned to four treatment arms; the tests use 33 baseline covariates. The data from [Auriol et al.](#page-32-1) [\(2020\)](#page-32-1) includes 1016 observations split in 148 clusters, with clusters randomly re-assigned to two "placebo" treatment arms and a control group; tests use 10 baseline covariates. For each test, we estimate the proportion of  $p$ -values or minimum  $q$ -values (out of 500) that are below a threshold t, for t ranging from 0.01 to 0.99 in steps of 0.01. We then use a fractional-polynomial prediction line to smooth the results. Figures [A.13](#page-51-0) and [A.14](#page-52-0) show how test size vary with the number of covariates and sample size for DGP 1 and DGP 2 respectively.



Figure A.11 Power of Omnibus Tests of Joint Orthogonality for DGP 1, with Multiple Treatment Arms

(c) 20% imbalanced variables, 0.15 SD



Notes: The figures show the results of simulations with a data generating process creating 50 independent variables that are normally distributed ∼N(0,1) and an independent treatment randomly assigned to half of observations. Imbalances are then created for a subset of the n variables and two treatment arms. In Panel (a), one variable is made imbalanced by adding 0.25 to treated observations. In Panel (b), 10% of variables are made imbalanced by adding 0.2 to treated observations. In Panel (c), 20% of variables are made imbalanced by adding 0.15 to treated observations. In Panel (d) 50% of variables are imbalanced, by adding 0.1 to treated observations. To produce each figure, a total of 100 simulated datasets of N observations are generated, for each  $N \in \{200, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000\}.$ We run pairwise  $t$ -tests for all  $n$  variables for each dataset and all possible comparisons between treatment arms. The minimum  $q$ -value is the minimum sharpened  $q$ -value obtained after applying the method o[fAnderson](#page-31-0)  $(2008)$  to the p-values from the pairwise t-tests. For each method, the light dots show the N-specific share of p-values or minimum q-values below 0.10, and the lines show fractional-polynomial predictions for  $N \in [200, 5000]$ . Results from multinomial logit with sampling-based inference are hidden as this approach has incorrect test size.

Figure A.12 Power of Omnibus Tests of Joint Orthogonality for DGP 2, with Multiple Treatment Arms  $\Lambda$  mm  $\alpha$ 



Notes: The figures show the results of simulations with a data generating process with N observations (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ) that are split into 100 clusters of equal size and 20 variables that are normally distributed and correlated within clusters (with an average intra-class correlation coefficient of 0.2). An independent treatment is equal to one in half of the clusters and zero otherwise. Imbalances are then created for a random subset of the n variables and two treatment arms. In Panel (a), one variable is imbalanced, by adding 0.25 to treated observations. In Panel (b), 10% of variables are imbalanced, by adding 0.2 to treated observations. In Panel (c), 20% of variables are imbalanced, by adding 0.15 to treated observations. In Panel (d), 50% of variables are imbalanced, by adding 0.1 to treated observations. To produce each figure, a total of 100 simulated datasets of N observations are generated, for each  $N \in 200, 500, 1000, 1500, 2000, 2500, 3000, 3500, 4000, 4500, 5000$ . For each N and each method, the light dots on each figure show the share of test statistics below 0.10. For each method, fractional-polynomial prediction lines are used to predict the power of tests for  $N \in [200, 5000]$ . Results from MANOVA and multinomial logit with sampling-based inference are hidden as these approaches have incorrect test size.

<span id="page-51-0"></span>Figure A.13 Size of Omnibus Tests of Joint Orthogonality for DGP 1 with Multiple Treatment Arms, as a Function of Sample Size





*Notes:* The figures show the results of simulations with a data-generating process that creates  $n$  independent variables that are normally distributed ∼N(0,1) (with  $n \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$ ) and a categorical variable identifying four randomly-assigned treatment arms with one quarter of observations in each. Simulated datasets of N observations are generated (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ). For each combination of n and N, we create 500 simulated datasets and run pairwise t-tests for all n variables for each dataset. The minimum  $q$ -value is the minimum sharpened  $q$ -value obtained after applying the method of [Anderson](#page-31-0) [\(2008\)](#page-31-0) to the p-values from the pairwise t-tests. For each  $N$ , the light dots show the n-specific share of test statistics below 0.10 and the lines show fractional-polynomial predictions for  $n \in [10, 100]$ . The multinomial logit did not converge for  $N=200$ .

<span id="page-52-0"></span>Figure A.14 Size of Omnibus Tests of Joint Orthogonality for DGP 2 with Multiple Treatment Arms, as a Function of Sample Size



(c) Mulitnomial logit, sampling-based inference (d) Mulitnomial logit, randomization inference and cluster-robust variance estimator and cluster-robust variance estimator



(e) minimum q-value, cluster-robust variance estimator

Notes: The figures show the results of simulations with a data-generating process with N observations (with  $N \in \{200, 500, 1000, 2000, 5000\}$ ) that are split into 100 clusters of equal size, and n variables that are normally distributed and correlated within clusters (with  $n \in \{5, 10, 15, 20\}$ , and an average intracluster correlation coefficient of 0.2). A categorical variable identifies four randomly-assigned treatment arms with one quarter of observations in each. For each combination of  $n$  and  $N$ , we create 500 simulated datasets and run pairwise t-tests for all  $n$  variables. The minimum  $q$ -value is the minimum sharpened  $q$ -value obtained after applying the method of [Anderson](#page-31-0) [\(2008\)](#page-31-0) to the p-values from the pairwise t-tests. The light dots show the share of p-values or minimum q-values below 0.10. For each  $N$ , fractionalpolynomial prediction lines are used to predict the size of tests for  $n \in [5, 20]$ .

	General Info.					Pairwise/Groupwise test				Joint Orthogonality $F$ -test			
Reference	Bal. test	$\overline{N}$	$\boldsymbol{n}$	$T+C$	Design	t	$\,F$	Nor.Diff	Inf.	J.F	Model	Inf.	VE
De Janvry et al. (2023)	Yes	2,854	11	$\overline{2}$	Clu	Yes	N <sub>o</sub>	No	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	clu.-rob
Ainsworth et al. (2023)	Yes	2,629	12	$\overline{2}$	Clu	Yes	N <sub>o</sub>	No	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	clu.-rob
Guryan et al. (2023)	Yes	Exp. 1: 2,633	19	$\overline{2}$	Ind	$\operatorname{Yes}$	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	clu.-rob
		Exp. 2: 2,710											
Macchi $(2023)$	Yes	Exp 1: 511	25	2	Ind	Yes	N <sub>o</sub>	N <sub>o</sub>	SB&RI	N <sub>o</sub>			
		Exp. 2: 238											
Buchmann et al. (2023)	Yes	26,408	18	3	Clu	Yes	N <sub>o</sub>	No	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	clu.-rob
Brock and De Haas (2023)	$\operatorname{Yes}$	2,054	6	$\boldsymbol{3}$	Ind	N <sub>o</sub>	N <sub>o</sub>	$\rm No$	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	HO
Hardy and McCasland (2023)	Yes	755	20	$\overline{2}$	Ind	Yes	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	HO
Baseler $(2023)$	Yes	497	20	$\overline{2}$	Ind	Yes	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>	$\overline{\phantom{0}}$		
Zárate $(2023)$	$\operatorname{Yes}$	6,147	23	3	Clu	Yes	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	Yes	ML	<b>SBI</b>	clu.-rob
Oh (2023)	$\operatorname{Yes}$	630	14	$\overline{2}$	Ind	$\operatorname{Yes}$	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>	$\overline{\phantom{a}}$		
Chakravorty, Dar, and Emerick (2023)	Yes	400	19	$\overline{2}$	Clu	N <sub>o</sub>	$\operatorname{Yes}$	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>			
Afrouzi et al. (2023)	$\rm No$												
Alan, Corekcioglu, and Sutter (2023)	Yes	1,988	19	$\overline{2}$	Clu	Yes	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>			
Cullen, Dobbie, and Hoffman (2023)	Yes	1,095	17	6	Ind	No	Yes	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>	$\overline{\phantom{a}}$	$\overline{\phantom{0}}$	
Gray-Lobe, Pathak, and Walters (2023)	Yes	4,125	14	$\overline{2}$	Ind	Yes	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	No	$\overline{\phantom{a}}$		
Banerjee et al. $(2023b)$	Yes	7,511	42	$\overline{2}$	Clu	Yes	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>	$\overline{\phantom{a}}$		
Battaglia, Gulesci, and Madestam (2023)	Yes	2,717	31	$\sqrt{2}$	Clu	Yes	N <sub>o</sub>	Yes	SBI $\&$ RI	N <sub>o</sub>			
Alesina, Miano, and Stantcheva (2023)	$\operatorname{Yes}$	22,506	8	3	Ind	N <sub>o</sub>	N <sub>o</sub>	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>	$\overline{\phantom{a}}$		
Jack et al. (2023)	Yes	1,840	$26\,$	6	Clu	N <sub>o</sub>	Yes	N <sub>o</sub>	<b>SBI</b>	N <sub>o</sub>	$\overline{\phantom{a}}$		
Christensen and Timmins (2023)	Yes	18,045	3	$\overline{2}$	Clu No	Yes	N <sub>o</sub>	<b>SBI</b>	$\rm No$				
Beaman et al. (2023)	Yes	6,807	9	3	Clu	$\operatorname{Yes}$	N <sub>o</sub>	$\rm No$	<b>SBI</b>	Yes	<b>OLS</b>	<b>SBI</b>	clu.-rob
Tungodden and Willén (2023)	N <sub>o</sub>							$\overline{\phantom{0}}$					

<span id="page-53-0"></span>Table A.1Literature Review of RCTs (2021-2023)



